**Heat Current Options**

So there are basically two different ways that we’ve defined the heat current – one via the energy current, and the other via the entropy current. The first way is as the *energy transfer* in the frame of reference where the particle current is zero, or equivalently, where the average particle velocity, **u**, is zero.



This is the definition we used when we discussed heat for the first time in the first Thermodynamics file. And this is the definition, practically, that we used when we defined heat current in the Thermodynamics Folder/Laws of Thermodynamics: Continuum file in the context of the energy balance equation,



and also in the Statistical Mechanics folder/Balance Equations file in that same context.



The definition from the Thermodynamics folder is a little more general, but they both congrue with the highlighted expression above, which is to say, we evaluate **j**ε in terms of external force **F**, chemical potential gradient ∇μ, and temperature gradient ∇T, and do the same for **j** itself. Then adjust **j**’s parameters to make **u** = 0 (and since **j** = n**u**, this makes **j** = 0 too) and fill these into the **j**ε expression. But this isn’t how it’s usually done. More often we say that the heat current is (T×) the entropy current, in the frame of reference where the particle velocity is zero, i.e., the current is zero:



where js = jq/T + s**u**, where **u** is the local average velocity, and jn = n**u**. This rather congrues with the operative definition we had in the Thermodynamics folder/Laws of Thermodynamics/Continuum (entropy balance), where we effectively made the definition:



and in the Statistical Mechanics/Balance Equations (Entropy) file, where we made the definition:



So these two approaches are clearly not identical across all possible expressions for f. Oh well. But maybe they are to first order in gradients and such? It seems that the predominant definition of **j**q is the one in terms of the entropy. In practical calculations, we need a definition though, that involves more energy terms than just kinetic energy and a field (this is all the stat mech sb definition is dealing with), and something more concrete than the one from the Thermodynamics file. So it is often done this way. We relate **j**s to **j**ε. This construction will also end up duplicating the results we got apropos heat transport and thermal conductivity from the NETD/continuum transport file. So suppose that s depends on the canonical variables ε and n (and not ℘ because we’ll presume to have impurities present which would foster a stationary equilibrium state, so ℘ wouldn’t be a variable in the equilibrium state as it’d always be 0). So we have identity (see Thermo folder/Thermo potentials file, and **u** = velocity, notation is getting annoying now).



We can add **j**q to both sides,



(recalling definition from 2nd law that **j**s = s**u** + **j**q/T). Now to first order, we can make the replacement p → π at our level of approximation (see Thermo folder/NETD continuum transport again, and π calculation in the Stat Mech / Boltzman conductivity file). So we can say:



where in the last line we also make replacement that εint → ε (ε = εint + ½m**u**2), which is acceptable to first order because **u** would be zero in equilibrium and so a first order small quantity in this non-equilibrium situation we have (remember we said --we have impurities, etc., which will work to disipate any non-zero convective motion). And so finally we have, recalling definition of **j**ε in the Balance Equations file:



But then, we’ll note that since **j** = n**u**:



So it appears that *fundamentally*, to first order, our expressions for jq *are* all equivalent. We typically use the ‘definition we actually use’ however, I think because it makes formulas come out nicer, typically (see thermal conductivity calculation for instance, in Stat Mech folder/electrical, thermal conductivity). But I think they’re probably all the same, to first order in gradients and things like that.

**Calculation of heat current via jqeb**

So in the Stat Mech / Balance equations file, we defined the heat current within the energy balance equation as:



I’d like to run a quick calculation to first order in gradients, forces, etc., to see if I get the same value for jq as I did in the Stat Mech / RTA conductivity file. So first, our distribution function, in the presence of a temperature, chemical potential, and electric potential gradient would be in the RTA approximation (to make it easy):



Where ε = k2/2m. So what is the jqeb? We are working just to first order, which means we can just go out to first order in **u**.



where we keep terms to first order in the gradients only, and ε = k2/2m. Recognizing the definitions of the pressure tensor, and the average energy, we can write:



Let’s speciallize to near T = 0. From the Stat Mech/Fermi Gas file, we have: **π** = (2/5)nεF**1**, while εeq = (3/5)nεF. So,



Now let’s fill in δf,



So to evaluate this we’d have to fill in **u**(r) = **j**(r)/n(r). In the presence of the aforementioned gradients, this is (see Stat Mech / RTA conductivity file):



Filling this in,



where,



Our expression doesn’t really simplify. We would expect that this is purely a function of the T gradient though. Right? So we’d expect the following equality,



Is this so? Well we used the small T approximation for εeq and πeq, so we must do the same for feq. This would be -∂nF/∂ε = δ(ε-εF). So then we have:



In 3D, ρF = 3n/2εF (see Condensed Matter / Metals / Free / Excitations). So,



So this part checks out! Now let’s look at M2. So,



Where in the last line, I’m taking τ to be constant, independent of energy. Now use the Sommerfield expansion (see Condensed Matter / Metals / Free / Thermal Equilibrium),



And we get:



where we evaluate at μ = εF. Now from the Condensed Matter / Metals / Free / Excitations file, we have:



So,



We’ll also use the result from Stat Mech / RTA conductivity,



Altogether then,



which is,



And this matches what we found in the Stat Mech / RTA Conductivity file, at least to this order.

**Calculation of heat current via jqsb**

So in the Stat Mech / Balance equations file, we defined the heat current within the entropy balance equation as:



which, according to the analysis above, is supposed to be equivalent to what we get from the energy balance definition of the heat current, jqeb, at least to first order in gradients, velocities, and such. So let’s check this out. Going out to first order,



What is ln(fleq) in the low T limit?



Well, I don’t really want to continue on with this calculation. Hopefully it would work out to be equivalent to jqeb.